Beverly Hills High School -- Honors Math Analysis -- Spring ‘16 -- Test #1 -- 90 points

On this and all following exams, give neat and complete answers, those that clearly show your understanding of the problem and its solution. In other words, show all your work. All problems are five points each unless specified otherwise. PENCILS ONLY.

For each of the following, state (for ten points total on each figure)

a) what conic section is represented
b) the general form for it
c) what is its center (or vertex if a parabola)
d) the coordinates of its foci
e) if it’s an ellipse, give length of semi-major axis; if it’s a hyperbola, give the asymptotes; if it’s a parabola, state the directrix and focal length
f) Then graph it -- please choose axes appropriately and sketch as neatly as you can.

1) $4x^2 + y^2 - 32x + 16y + 124 = 0$

2) $44 - 4y^2 - 72x - 16y + 12x^2 = 0$

3) $-x^2 - 4y - 4x = 0$

4) $x^2 + y^2 - 22y + 14x + 89 = 0$

5) Identify the conic: $3x^2 - 2xy - 5x + 6y - 10 = 0$. Then solve for y. Then state its domain and range. 5 points. (be careful!!)

6) Set up, but do not calculate, an expression for the eccentricity of the elliptical orbit of Halley's comet if its orbit has a major axis of 36.18 astronomical units (AU) and a minor axis of 9.12 AU. 5 points.

7) For ten points, find the angle of rotation, α, for the x'-y' axes relative to the x-y axes and find the coordinates of the point (1, 2) in the rotated x'-y' plane for this conic section:

$$2\sqrt{3}\,x^2 + 3xy + \sqrt{3}\,y^2 - 5x + 2y + 8 = 0$$

8) For ten more, what is the geometric definition of (using the notion of loci)

a) a parabola ____

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b) an ellipse ____

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c) a hyperbola ____

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9) Find the equation in standard form for the hyperbola whose center is at (1, -4), vertices at (-4, -4) and (6, - 4) and asymptotes y = ±(x - 1) - 4.

10) Find the equation in standard form for the parabola with focus (-5,3) and vertex (-5, 6).

11) One of the problems that attracted the interest of the early Greek mathematicians is the duplication of the cube. In this case, “duplication” does not mean making an exact copy. The problem is to find the side of a cube whose volume is double that of a given cube.

Well, it turns out the Greeks only had compass and straightedge to solve this problem. Alas, with these restrictions, there is no way to solve the problem. However it can be solved with the use of conic sections.

Algebraically the duplication of the cube can be written as $x^3 = 2a^3$ where a is the side of the given cube.

In 420 BC, Hippocrates made progress in this problem by setting a system of proportions:

$a:x = x:y = y:2a$.

a) Use these proportions to write a system of quadratic equations and arrive at the equation given above.

b) In the 1600’s, Rene Descartes pointed out a different solution. He used the system

$x^2 = ay$ and $x^2 + y^2 = ay + bx$. In his solution, $b = 2a$. Graph this system of conics below for $a = 4$ and estimate the value of the solution x, where the graphs intersect.