

Beverly Hills High School -- AP Calculus BC -- Test #6b -- Sections 6.4-6.6 -- 80 points

SHOW ALL YOUR WORK. Convince me (and the AP examiners) that you know exactly what you are talking about. Neatness is good, too...and calculators are okay...

- 1) It is written in the Bible that Jacob and his family traveled to Egypt with just seventy people. When they left 400 years later, they were 2,000,000 strong. Given that their rate of population growth was proportional to the number in the population, derive an expression for $P(t)$ and state how many people were present after only fifty years.
- 2) A newly discovered radioactive isotope, Nandinium-88, was found to have a half-life of fifty years. Find k , write an expression for $A(t)$, and determine how long it takes for a specific amount to reduce to just 10% of its original amount.
- 3) Using series expansions to three terms each (and your calculator), give an estimate for the value $\frac{e^3}{\ln 5}$. Then give a calculated value to three decimal places and calculate the percentage error in your estimate.

4) A population of drosophila (fruit flies) are present in a culture chamber. The chamber can hold no more than 5000 flies. If we assume that the relative growth rate is proportional to $1 - P/M$ and has a positive proportionality constant k , we have $\frac{dP}{dt} = \frac{k}{M}P(M - P)$. Given this, derive the logistic growth model,

$P(t)$. Fifteen points. For five more, given $k = 0.05$ and starting population of 100, determine how many days will pass until there are 1000 fruit flies present in the chamber.

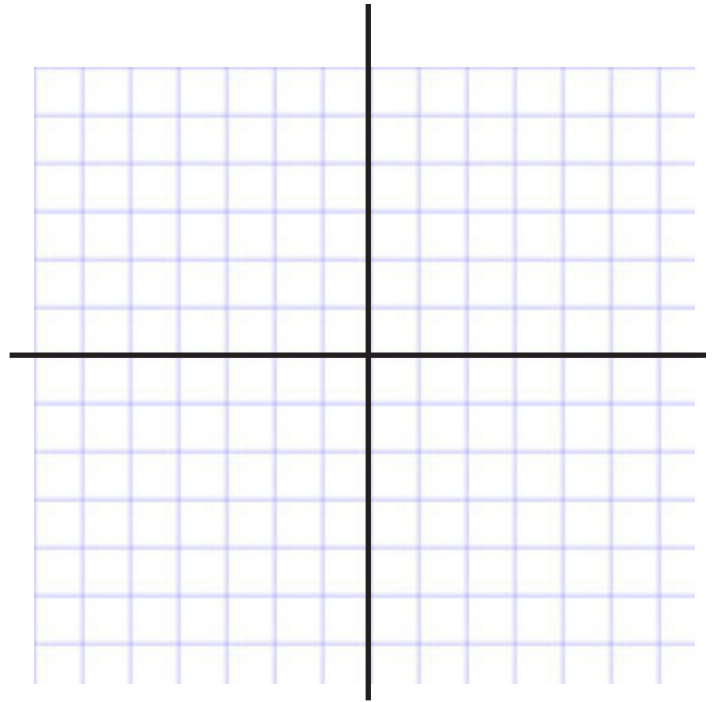
- 5) A hot ingot of silver is set in a room whose ambient temperature is $20\text{ }^{\circ}\text{C}$. The hot ingot is originally at a temperature of $1800\text{ }^{\circ}\text{C}$. After 20 minutes, it has cooled to $1000\text{ }^{\circ}\text{C}$. Using Newton's Law of Cooling, determine how long it will take to cool to $100\text{ }^{\circ}\text{C}$.

6) Evaluate this integral completely: $\int_0^1 \frac{16}{x^2 \sqrt{4-x^2}} dx$

7) Using the graph at the right and small lines about this size $|$, compose a slope field and sketch out three solution curves for

$$\frac{dy}{dx} = x - y^2$$

The slope field must contain at least twenty small lines.



8) EXTRA CREDIT. All or nothing. Guessing is okay. Five points.

A friend of mine visited a scientific outpost in Antarctica last year. He said the day he got there the temperature was -5°C . I said, "Sounds cold." He said, "Yeah it was, but the day before it was twice as cold." What was the temperature the day before?